Adverse Effect of Humans on Animals in Water Bodies: A Mathematical Study

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ABSTRACT

A mathematical analysis is presented to demonstrate the negative impact of humans on the survival of animals in bodies of water. Also, the effect of pollutants and their interaction with bacteria is considered. One of the majorities of animals in the aquatic kingdom is of course the fish population. Animal populations are considered to be fish. Fish survival is difficult due to human activities like sewage discharge and other human waste. Therefore, the combined effect of the four variables, namely, human population, organic pollutants, bacteria, and fish population is studied.

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1. Introduction

Water is essential for the survival of humans and all other animals on earth. It is critical that the human population's behavior change to be good and responsible in order for all aquatic animals to survive [1].There is an imbalance between humans and the aquatic world due to a scarcity of fresh water.Water contamination is also a major problem for aquatic animals' survival [2]. Rapid growth of the human population is also a main cause of the pollution of water bodies [3]. Also, the groundwater is also heavily polluted due to the presence of toxic substances and inorganic pollutants [4]. Irrigation practices are also increasing as a result of pollution. Overall, the quality of natural resources is depleting day by day due to human population growth.

Water quality degradation has contributed to scarcity and created barriers to its use for humans and animal populations on the surface and in bodies of water [5].The negative effects of human-related activities have largely destroyed almost all water bodies where humans live [6]. Due to this, aquatic animals are also facing difficulties for their survival. There is also a high load of domestic waste in nearby water bodies, which has caused enormous damage to the health of aquatic animals [6–9]. Due to a lot of domestic and industrial waste reaching aquatic bodies, there is an increased concentration of bacteria, viruses, and other harmful organisms that cause harm to the health of the human and aquatic populations [10–12].

2. Mathematical System:

Consider an aquatic environment to see the adverse effect of humans on fish survival. For positive time t, let N be the density of humans, T be the density of organic pollutants, B be the bacteria density, and F be the density of fish in an aquatic environment. The growth rate of humans is assumed to follow the logistic trend. The rate of pollutant transfer into aquatic bodies due to human domestic sewage is taken into account. Bacteria degrade pollutants in the aquatic body to some extent. The bacteria experience natural death in aquatic environments. It is assumed here that fish survive easily because of the presence of dissolved oxygen in the water.

Fish growth is thought to be proportional to the density of the body of water. Organic pollutants present in the body of water deplete the dissolved oxygen level in the body of water. Fish death is regarded as a natural occurrence, as is human hunting. Considering all the factors mentioned, the system is given by the following nonlinear equations:

$$\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right) + \theta\alpha FN,$$

$$\frac{dT}{dt} = Q + \delta N - \alpha_0 T - KTB\ ,$$

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$\frac{dB}{dt} = \lambda KTB - \theta_1 B$
(1)

$$\frac{dF}{dt} = \theta F - \alpha FN - \theta_0 F$$

We analyse the system (1) with the following conditions:

N(0) ≥ 0, T(0) > 0, B (0) ≥ 0, F(0) ≥ 0.

In the system (1), r is the intrinsic growth rate of humans, and E is the carrying capacity when no fish are present. Humans emit organic pollutants at a rate δ, and other human emissions are referred to as Q. Further pollutants settle at the bottom after some time. By applying some techniques, the organic pollutants can be removed from the body of water. The natural depletion is included in the system at a rate of $\alpha_{0.}$

Furthermore, bacteria aid in the decomposition of pollutants at rate E. Due to the decomposition of pollutants by bacteria the growth of bacteria increases correspondingly. The bacteria experience natural depletion at a rate of $\theta_{1.}$

Furthermore, fish growth accelerates at a rate of θ, Natural fish death is also included at a rate of θ_0. Human fishing reduces fish density at a steady rateα.

3. Mathematical analysis of the system

3.1 *Equilibrium analysis*

The system (3.1) exhibits three non-negative situations i.e., equilibria as listed following

(i) The equilibrium $E_0\left(0,\ \frac{Q}{\alpha_0}, 0, 0\right)$, always exist.

(ii) The equilibrium $E_1(N_1,\ T_1, B_1, 0)$,where

$N_1 = K, T_1 = \frac{\theta_1}{\lambda K}, B_1 = \frac{\lambda K(Q+\delta N_1 - \alpha_0 T_1)}{K\theta_1}$

exist if $Q + \delta K > \frac{\alpha_0 \theta_1}{\lambda e}$

(iii) The equilibrium

$E_2(N_2,\ T_2, B_2, F_2)$ exists if the condition given below is feasible

$\frac{\theta+\theta_0}{\alpha K} > 1,$

$N_2 = \frac{\theta+\theta_0}{\alpha},\qquad T_2 = \frac{\theta_1}{\lambda K}, B_2 = \frac{\lambda K(Q+\delta N_1 - \alpha_0 T_1)}{K\theta_1},$
$F_2 = \frac{\theta+\theta_0 - \alpha K}{\theta \alpha^2 K},$

Now we present the proof of equilibria existence

Proof: For finding all components of all the equilibria of the system (3.1). Set all the rate of change variable zero to get the following system of algebraic equations

$rN\left(1 - \frac{N}{K}\right) + \theta\alpha FN = 0$ (1)

$Q + \delta N - \alpha_0 T - KTB = 0,$ (2)

$\lambda KTB - \theta_1 B = 0,$ (3)

$\theta F - \alpha FN - \theta_0 F = 0$ (4)

From equation (3), we have

$N = 0\ or\ \left(1 - \frac{N}{K}\right) + \theta\alpha F = 0,$

From equation (5), we have

B = 0 $or \lambda KT - \theta_1 = 0,$

From equation (6),

$\theta - \alpha N - \theta_0 = 0,$

From equations (5), (6) and (7), we obtain the components of first equilibrium E_0

$E_0\left(0,\ \frac{Q}{\alpha_0}, 0, 0\right),$

For the second equilibrium $E_{1,}$

Taking F=0, then we get equilibrium components as follows

$N_1 = K, T_1 = \frac{\theta_1}{\lambda K}, B_1 = \frac{\lambda K(Q+\delta N_1 - \alpha_0 T_1)}{K\theta_1}$

$E_1(N_1,\ T_1, B_1, 0)$, exist if $Q + \delta K > \frac{\alpha_0 \theta_1}{\lambda e}$

For the components of equilibrium E_2

Taking all dynamic variables nonzero to get

$N_2 = \frac{\theta+\theta_0}{\alpha},\ T_2 = \frac{\theta_1}{\lambda K}, B_2 = \frac{\lambda K(Q+\delta N_1 - \alpha_0 T_1)}{K\theta_1},\ F_2 = \frac{\theta+\theta_0 - \alpha K}{\theta\alpha^2 K},$

The equilibrium $E_2(N_2,\ T_2, B_2, F_2)$ exists if the condition given below is feasible

$\frac{\theta+\theta_0}{\alpha K} > 1,$

4. Stability Analysis

The system (3.1) has been also analysed in the sense of local stability. By using Routh Hurwitz matrix stability analysis of all the equilibria has been performed by checking the sign of eigenvalues of the Jacobian matrix evaluated at the respective equilibrium values. These results are also verified numerically, will be discussed in the section of numerical analysis.

The Jacobian matrix of the system (3.1) as follows

$$J = \begin{bmatrix} a_{11} & 0 & 0 & a_{14} \\ a_{21} & a_{22} & a_{23} & 0 \\ 0 & a_{32} & a_{33} & 0 \\ a_{41} & 0 & 0 & a_{44} \end{bmatrix}$$

$$a_{11} = r - \frac{2N}{K} + \theta\alpha F,\ \ a_{14} = \theta\alpha N,\ \ a_{21} = \delta,\ \ a_{22} = -\alpha_0 - KB,\ \ a_{23} = -KT,$$

$$a_{32} = \lambda KB,\quad a_{33} = \lambda KT - \theta_1, a_{41} = -\alpha F,\ \ a_{44} = \theta - \theta_0 - \alpha N$$

Following results are obtained:

(1) The equilibrium E_0 is locally asymptotically stable if $\theta < \theta_0 \& r < 0$

(2) The equilibrium E_1 is locally asymptotically stable if $\theta - \theta_0 < \alpha N_1, r < \frac{2N_1}{K}, \ -\theta_1 + \lambda K T_1 < 0$

(3) The equilibrium E_2 is locally asymptotically stable always.

Proof: Let μ be an eigenvalue of the Jacobian matrix of the system (3.1).

(1) Local asymptotic stability conditions of the equilibrium E_0 may be obtained by evaluating eigen values of Jacobian matrix at E_0 which is as follows

$$J_{E_0} = \begin{bmatrix} r & 0 & 0 & a_{14} \\ \delta & -\alpha_0 & -KQ/\alpha_0 & 0 \\ 0 & a_{32} & -\theta_1 & 0 \\ a_{41} & 0 & 0 & -(\theta - \theta_0) \end{bmatrix}$$

The characteristic equation is follows:

$(\mu - r)(\mu + \alpha_0)(\mu + \theta_1)(\mu - \theta + \theta_1)$=0,

It is very clear that the eigenvalues ofJ_{E_0} are $r, -\alpha_0, -\theta_1, \theta - \theta_0$.

By applying stability criterion, we conclude that the equilibrium E_0 is locally asymptotically

Stable if$\theta < \theta_0 \& r < 0$.

(2) Local asymptotic stability conditions of the equilibrium E_1 may be obtained by evaluating eigen values of Jacobian matrix at E_1 which is as follows

$$J_{E_1} = \begin{bmatrix} r - \dfrac{2N_1}{K} & 0 & 0 & \theta \alpha N_1 \\ \delta & -\alpha_0 - KB_1 & -KT_1 & 0 \\ 0 & \lambda K B_1 & \lambda K T_1 - \theta_1 & 0 \\ 0 & 0 & 0 & (\theta - \theta_0 - \alpha N_1) \end{bmatrix}$$

The characteristic equation is follows:

$\{(\mu - (\theta - \theta_0 - \alpha N_1)\}\left(\mu - \left(r - \frac{2N_1}{K}\right)\right)(\mu + \alpha_0 + KB_1)(\mu + \theta_1 - \lambda K T_1)$=0,

It is very clear that the eigenvalues of$J_{E_0}$are

$\theta - \theta_0 - \alpha N_1, r - \frac{2N_1}{K}, \ \alpha_0 + KB_1, -\theta_1 + \lambda K T_1.$

By applying stability criterion, we conclude that the equilibrium E_1 is locally asymptotically

Stable if$\theta - \theta_0 < \alpha N_1, r < \frac{2N_1}{K}, \ , -\theta_1 + \lambda K T_1 < 0.$

(3) The equilibrium E_2 is always locally asymptotically stable without any condition. The eigenvalues of the Jacobian matrix evaluated at the equilibrium E_2 are found to be negative. Therefore, we can conclude from Routh-Hurwitz criterion that the equilibrium E_2 is locally asymptotically stable. Also, the eigenvalues are negative without any conditions.

5. Numerical Analysis

For numerical analysis of the system (3.1), numerical simulation has been performed to solve the system of first order nonlinear differential equations by using MATLAB software build in function *ode45* has been used to get numerical results.

Parameter values used in numerical methods of the system (3.1)

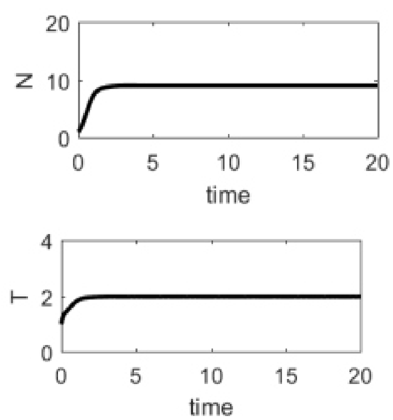
Parameter	Value
r	**0.01**
α_0	**2**
$\boldsymbol{\theta}$	**1.001**
α	**0.67**
Q	**3**
δ	**0.02**
λ	**0.5**
θ_1	**0.002**
θ_0	**0.06**

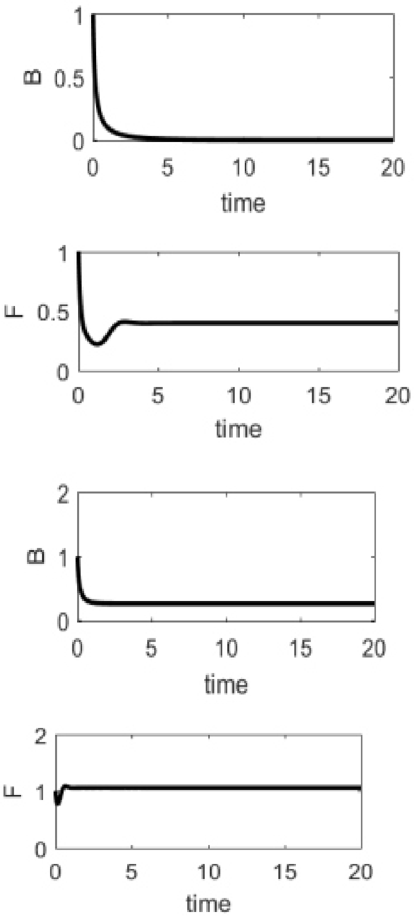
For this collection of parameter values all the analytical conditions of existence and stability are satisfied. The eigenvalues obtained of the Jacobian matrix as follows

0.2752 + 0.9900i, −0.2752 − 0.9900i, −0.1003, −0.0201, −3.2400.

These negative eigenvalues or negative eigenvalues with real part shows that the equilibrium E_2 is asymptotically stable.

The following figures are plotted to see the adverse effect of human population on the aquatic animals in water bodies. The figure shows that the dynamic variables are stable.





6. Conclusion

In this paper, we have considered the dynamics of human populations, organic pollutants, bacteria and fish populations using first-order differential equations, which are of a nonlinear nature. The analytical and numerical results have been presented, which show the adverse effect of human population growth on the fish population. The result from analysis and simulation shows that all the equilibria are locally asymptotically stable. It is very important to control the inflow of organic pollutants into bodies of water to minimize the harmful effects on the animal population of those bodies.

7. References

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