Kantowski-Sach String Cosmological Stability with Electromagnetism in f(R,T) theory

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ABSTRACT

In this article, we intended to examine the Kantowski-Sach string cosmological model in the electromagnetic field with f(R,T) theory of gravity, where R shows Ricci Scalar and trace T shows stress energy tensor. At this time we proposed mixed energy momentum tensor as $T_{ij} = S_{T_{ij}} + E_{T_{ij}}$, where $E_{T_{ij}}$ shows energy momentum tensor in the electromagnetic field and $S_{T_{ij}}$ shows energy momentum tensor in the string cloud. In this condition, we have to study the stability of the cosmological model and we find the exact solution in two cases of field equation in f(R,T) theory. We ensure the universe is stable or not. Some physical properties are discussed in detail in the investigated model.

Keywords: *Kantowsk-Sach cosmology, Cosmic String Theory, f(R,T) gravitational theory, and Electromagnetic field.*

***2020 Mathematics Subject Classification**: 97M50*

1. Introduction

"As per Riess et al. (1998), the discovery of the present universe is expanding and accelerating in modern cosmology. As per Perlmutter et al. (1999), in the high red-shift supernovae experiments, the universe acceleration and expansion have been authenticated in late time. As per Bennet et al. (2003), in the large and negative pressure, our universe is dominated by strange cosmic fluid. As Per Brans & Dicke (1961); Saez & Ballester (1986), cosmologists are operational in alternative theories of gravitation. Kantowski-Sachs (1966) gave solutions for dust space time, Collins (1977) for perfect fluid, Barrow et al. (1997) for scalar fields, Gergely (1999) for anisotropic fluid, and Gergely (2002) for exotic fluid models. Tikekar et al. (1992); Thorne (1967) and Roy et al. (1978) are also studied the magnetic field of string cosmological models. Xing-Xang (2004), in the existence of Bulk Viscosity & magnetic field, investigated symmetric (LRS) Bianchi first type cosmological model. Wang (2005), In the Kantowski-Sachs space time, analyzed string cosmological models of bulk viscosity As per Takahashi (2010), in matter Lagrangian density L_m was anticipated the modified f (R) theory of gravity, investigated an explicit coupling of Ricci scalar R. After that, Harko et al. (2011), the Ricci scalar R and the stress energy tensor T, proposed f (R, T) modified theories of gravitation. Recently, In Lyra's geometry, Chaubey (2012) analyzed the Kantowski-Sachs model field in the perfect fluid. Adhav (2012) and Reddy et al. (2012), in the $f(R,T)$ theory of gravity, they studied different cosmological models. Kantowski-Sachs cosmological modal contains two symmetry properties first is spherical symmetry and the second is invariance under spatial translations. In the f(R) theory of gravity, Katore et al. (2015) investigated the Stability of the Kaluza–Klein cosmological model with holographic dark energy. Katore et al. (2016), investigated the FRW metric with the constant deceleration parameter and analyzed that domain walls vanished at a large time, in the early era the universe could exist and the universe is stable".

Therefore, we inspired on top of investigations and discussion. We discuss the physical stability string cosmological model of Kantowski-Sach in the electromagnetic field. We obtain the field equation in f(R, T) theory and found the exact solution in two ways. First, we consider the relation $A = B^n$. Second, we consider the deceleration parameter for Kantowski-Sach is constant.

This paper is structured in six sections. In the first section, we discuss the introduction. Second Section, we show the dynamical field equations in f(R, T) theory of gravitation. Third Section, we show the Kantowski-Sach string cosmological model. Fourth Section, we discuss the solution of the related field equations in two ways. First, we consider the relation $A = B^n$. Second, we consider the deceleration parameter for Kantowski-Sach

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constant. Fifth Section, we discuss the stability of the universe and the result of its physical properties. Sixth Section contains the conclusion.

2. F(R, T) gravitational theory:

Harko et al. (2007) given f (R, T) modified gravitational theory model. In the variational principle type Einstein field equation as defined:

$$S = \frac{1}{16\pi}\int \sqrt{-g}(f(R,T) + L_m)d^4x \quad (1)$$

In the f (R, T) function, R shows Ricci scalar and T shows energy momentum tensor. They have given three classes models the first class is $f(R,T) = 2f(T) + R$, the second class is $f(R,T) = f_2(T) + f_1(R)$, and the third class is $f(R,T) = f_2(R)f_3(T) + f_1(R)$. In General, the first class has more attention but we have studied the second cosmological consequences class.

$$f(R,T) = f_1(R) + f_2(T) \quad (2)$$

Here, the field equation in the gravitational field is defined as:

$$f_1'(R)R_{ij} - \frac{1}{2}f_1(R)g_{ij} + (g_{ij}\,\Box - \nabla_i\nabla_j)$$

$$f_1'(R) = 8\pi T_{ij} + f_2'(T)T_{ij} + (P\,f_2'(T) + \frac{1}{2}f_2(T))g_{ij} \quad (3)$$

we assume the functions $f_2(T) = \lambda_2 T$ & $f_1(R) = \lambda_1 R$ and $\lambda_1 \neq \lambda_2$ after putting the value in eq^n(2) we got $f(R,T) = \lambda_1 R + \lambda_2 T$. After putting this condition then the eq^n(3) found as:

$$\lambda_1 R_{ij} - \frac{1}{2}g_{ij}\lambda_1 R + \lambda_1(g_{ij}\,\Box - \nabla_i\nabla_j) = 8\pi T_{ij} + \lambda_2 T_{ij} + \left(\lambda_2 P + \frac{1}{2}\lambda_2 T\right) g_{ij} \quad (4)$$

Assuming $(g_{ij}\,\Box - \nabla_i\nabla_j)\lambda_1 = 0$, we obtain

$$R_{ij} - \frac{1}{2}Rg_{ij} = \left(\frac{8\pi + \lambda_2}{\lambda_1}\right)T_{ij} + \frac{\lambda_2}{\lambda_1}(P + \frac{1}{2}T)g_{ij} \quad (5)$$

3. Kantowski-Sachs Metric equation:

The Kantowski-Sachs metric is defined as:

$$ds^2 = -dr^2A^2 - (d\theta^2 + \sin^2\theta\, d\varphi^2)\, B^2 + dt^2 \quad (6)$$

Here A and B are only time functions.

Here the matter's energy momentum tensor is defined as:

$$T_{ij} = S_{T_{ij}} + E_{T_{ij}} \quad (7)$$

Where $S_{T_{ij}}$ shows the cloud string energy momentum tensor and $E_{T_{ij}}$ shows the electromagnetic field energy momentum tensor. The cloud string energy momentum tensor is defined as:

$$S_{T_{ij}} = \rho\, u_i u_j - \lambda\, x_i\, x_j \quad (8)$$

Here $u_i u^j = -x_i x^i = -1$, $x_i u_i = 0$

Where ρ show the cloud string rest energy density of the particles, λ show the string tension density of, u_i show the four velocities of cloud string and x_i show the direction of anisotropy. The magnetic field is used along φ direction in the commoving coordinates. So the electromagnetic field tensor F_{ij} is F_{12} as a non-vanishing component. We taken F_{12} = constant = J (say) = F_{21}

$$F^{12} = g^{1\alpha}g^{1\beta}F_{\alpha\beta} \quad (9)$$

The Electromagnetic energy momentum tensor:

$$E_{T_{ij}} = \frac{1}{4\pi}\left[-F_{is}F_{jp}\,g^{sp} + \frac{1}{2}g_{ij}F_{sp}\,F^{sp}\right] \quad (10)$$

Here, F_{ij} shows the electromagnetic field tensor as:

$$F_{ij} = \varphi_{i,j} - \varphi_{j,i} \quad (11)$$

So the Maxwell equation

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 \quad (12)$$

Here F_{12} = J (constant) and other all components are zero. Eq^n 8 and 10 have the non-vanishing components of $S_{T_{ij}}$ and $E_{T_{ij}}$ found as:

$$S_{T_1^1} = S_{T_2^2} = 0\,, S_{T_3^3} = -\lambda\,,$$

$$S_{T_4^4} = -\rho \quad (13)$$

$$E_{T_1^1} = E_{T_2^2} = -E_{T_3^3} = -E_{T_4^4} = -\frac{H^2}{8\pi A^2B^2} \quad (14)$$

The Einstein field equation as

$$G_j^i = R_j^i - \frac{1}{2}\delta_i^j R \quad (15)$$

Putting the eq^n (5) we get

$$G_j^i = \frac{(8\pi+\lambda_2)T_j^i}{\lambda_1} + \frac{\lambda_2}{\lambda_1}(P + \frac{T}{2})\delta_i^j \quad (16)$$

The Field eq^n (6) for the metric is as follows:

$$\frac{\dot{B}^2}{B^2} + \frac{2\ddot{B}}{B} + \frac{1}{B^2} = \frac{J^2}{8\pi A^2B^2}\frac{(8\pi+\lambda_2)}{\lambda_1} - \left(P - \frac{\lambda}{2} - \frac{\rho}{2}\right)\frac{\lambda_2}{\lambda_1} \quad (17)$$

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = \frac{J^2}{8\pi A^2B^2}\frac{(8\pi+\lambda_2)}{\lambda_1} - \left(P - \frac{\lambda}{2} - \frac{\rho}{2}\right)\frac{\lambda_2}{\lambda_1} \quad (18)$$

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = \left(\lambda - \frac{J^2}{8\pi A^2B^2}\right)\frac{(8\pi+\lambda_2)}{\lambda_1} - \left(P - \frac{\lambda}{2} - \frac{\rho}{2}\right)\frac{\lambda_2}{\lambda_1} \quad (19)$$

$$\frac{\dot{B}^2}{B^2} + \frac{2\dot{A}\dot{B}}{AB} + \frac{1}{B^2} = \left(\rho - \frac{J^2}{8\pi A^2B^2}\right)\frac{(8\pi+\lambda_2)}{\lambda_1} - \left(P - \frac{\lambda}{2} - \frac{\rho}{2}\right)\frac{\lambda_2}{\lambda_1} \quad (20)$$

4. Solution of the field equation:

4.1 *Case I*

In this section, the solution of the field equations 24, 25, 26, and 27 are found in the presence of a cosmic string in the electromagnetic field. Here four equations contain seven unknowns as A, B, λ_1, λ_2, λ, P, and ρ. To find the complete solution to the system we required one extra condition. Therefore we assume a relation $A = B^n$

Here we took n as an arbitrary constant

Subtract eq^n (18) from eq^n (19) we get

$$\frac{(8\pi+\lambda_2)}{\lambda_1}\left(\lambda - \frac{J^2}{8\pi A^2 B^2} - \frac{J^2}{8\pi A^2 B^2}\right) = 0$$

$$\lambda = \frac{2J^2}{8\pi A^2 B^2} \quad (21)$$

Subtract eq^n (17) from eq^n (20) we get

$$\frac{2\dot{A}\dot{B}}{AB} - \frac{2\ddot{B}}{B} = \frac{(8\pi+\lambda_2)}{\lambda_1}\left(\rho - \frac{J^2}{8\pi A^2 B^2}\right) \quad (22)$$

Subtract eq^n (17) from eq^n (18) we get

$$-\frac{2\ddot{B}}{B} - \frac{1}{B^2} - \frac{\dot{B}^2}{B^2} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = 0 \quad (23)$$

Substitute $A = B^n$ we get

$$\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2}(n+1) - \frac{1}{(n-1)}\frac{1}{B^2} = 0$$

We can write as

$$\dot{B}^2 = \frac{1}{(n^2-1)} + D\,B^{-2n-2}$$

This equation has a first integral form of B as

$$\int\left[\frac{1}{(n^2-1)} + D\,B^{-2n-2}\right]^{-1/2} dB = \pm(t -) \quad (24)$$

Here D & □ are the integration constants. This equation cannot be solved for any values of n. here we can find the exact solution only for $n = 0, -2$ & $-3/2$.

For $n = 0$

$$\int\left[\frac{1}{-1} + D\,B^{-2}\right]^{-1/2} dB = \pm(t -)$$

After integration we get

$$B = D - (t - + k)^2]^{1/2}$$

Here k is an arbitrary constant.

Therefore, this model is a contracting model of the universe so we have no physical interest.

For $n = -2$

$$\int\left[\frac{1}{3} + D\,B^2\right]^{-1/2} dB = \pm(t -)$$

After integration we get $B = \frac{1}{\sqrt{3D}}\,\mathrm{Sinh}[\sqrt{D(t -)}]$

Therefore, this model is a contracting model of the universe so we have no physical interest.

For $n = -3/2$

$$\int\left[\frac{4}{5} + D\,B\right]^{-1/2} dB = \pm(t -)$$

After integration we get

$$B = \left[\frac{D}{4}(t -)^2 - \frac{4}{5D}\right] \quad (25)$$

Here, $A = B^n$ put $n = -3/2$

$$A = \left[\frac{D}{4}(t -)^2 - \frac{4}{5D}\right]^{-3/2} \quad (26)$$

Putting the values of A and B in ds^2, we get

$$ds^2 = dt^2 - \left[\frac{D}{4}(t -)^2 - \frac{4}{5D}\right]^{-3} dr^2 - \left[\frac{D}{4}(t -)^2 - \frac{4}{5D}\right]^2 (d\theta^2 + \mathrm{Sin}^2\theta\, d\varphi^2)$$

So it is clear that the eq^n (24) cannot be solved for any values of n and D in cases I, II, and III. So, we have worked on only $n = -3/2$.

Physical Properties

4.1.1 The tensor density of the cloud string as:

$$\lambda = \frac{2J^2}{8\pi A^2 B^2}\ \lambda = \frac{2J^2}{8\pi\left[\frac{D}{4}(t-)^2 - \frac{4}{5D}\right]^{-3}\left[\frac{D}{4}(t-)^2 - \frac{4}{5D}\right]^2}$$

$$\lambda = \frac{J^2}{4\pi}\left[\frac{D}{4}(t -)^2 - \frac{4}{5D}\right] \quad (27)$$

Therefore, if $J \neq 0$ then the tensor density of the cloud string tends to be constant at $t \to$ and if $J = 0$ then the tensor density is zero.

4.1.2 Rest energy density:

From the eq^n (22)

$$\frac{2\dot{A}\dot{B}}{AB} - \frac{2\ddot{B}}{B} = \left(\rho - \frac{2J^2}{8\pi A^2 B^2}\right)\frac{(8\pi+\lambda_2)}{\lambda_1}$$

$$-3\frac{\dot{B}^2}{B^2} - \frac{2\ddot{B}}{B} = (\rho - \lambda)\frac{(8\pi+\lambda_2)}{\lambda_1}$$

Substitute the value of B from the equation (25) we get

$$\rho = -\frac{\lambda_1}{(8\pi+\lambda_2)}\left[\frac{\frac{3D^2}{4}(t-)^2}{\left[\frac{D}{4}(t-)^2 - \frac{4}{5D}\right]^2} + \frac{D(t-)}{\left[\frac{D}{4}(t-)^2 - \frac{4}{5D}\right]}\right] + \frac{J^2}{4\pi}\left[\frac{D}{4}(t -)^2 - \frac{4}{5D}\right] \quad (28)$$

Therefore, the Rest of energy density of the cloud string tends to be constant at $t \to$ and if we have taken $\lambda_1 = 1, \lambda_2 = \pi, D = 1, = 1, J = 1$, then the Rest energy density of cloud string tends to infinite as t is large.

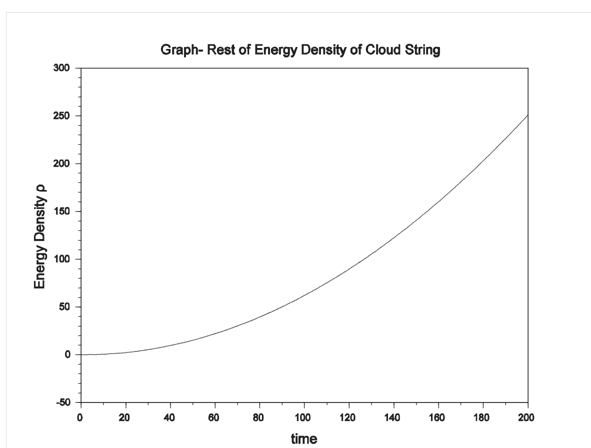


Fig. 1

4.1.3 Pressure P:

From the eq^n (17)

$$\frac{\dot{B}^2}{B^2} + \frac{2\ddot{B}}{B} + \frac{1}{B^2} = \lambda\ \frac{(8\pi + \lambda_2)}{\lambda_1} - \left(P - \frac{\lambda}{2} - \frac{\rho}{2}\right)\frac{\lambda_2}{\lambda_1}$$

Substitute the values of B from (25), λ from (27), and ρ from (28) we get

$$P = -\frac{\lambda_1}{2(8\pi + \lambda_2)}\left[\frac{D(t-)}{[\frac{D}{4}(t-)^2 - \frac{4}{5D}]} + \frac{\frac{3D^2}{4}(t-)^2}{[\frac{D}{4}(t-)^2 - \frac{4}{5D}]^2}\right] + \frac{(8\pi + 3\lambda_2)}{\lambda_2}\frac{J^2}{4\pi}\left[\frac{D}{4}(t-)^2 - \frac{4}{5D}\right] - \frac{\lambda_1}{\lambda_2}\left[\frac{\frac{D}{2}(t-)}{[\frac{D}{4}(t-)^2 - \frac{4}{5D}]} + \frac{\frac{D^2}{4}(t-)^2 + 1}{[\frac{D}{4}(t-)^2 - \frac{4}{5D}]^2}\right] \quad (29)$$

Therefore, the Pressure of the cloud string tends to be constant at $t \rightarrow$ and if we have taken $\lambda_1 = 1, \lambda_2 = \pi, D = 1, = 1, J = 1$ then the Pressure of the cloud string tends to infinite as t is large.

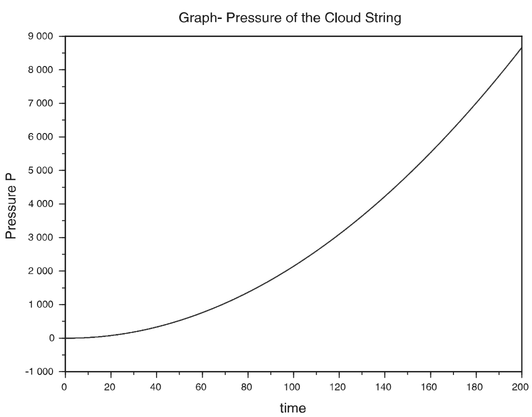


Fig. 2

4.1.4 Spatial Volume:

$$V = \sqrt{-g}$$

$$V = AB^2 \sin\theta$$

$$V = B^{1/2}\sin\theta \quad (30)$$

$$V = [\frac{D}{4}(t-)^2 - \frac{4}{5D}]^{1/2}\sin\theta$$

At t =

$$V = \sin\theta\,[-\frac{4}{5D}]^{1/2}$$

Since $\theta = \pi/2$ then

$V = [-\frac{4}{5D}]^{1/2}$ = constant at $t \rightarrow$

Therefore, the spatial volume is constant at $t \rightarrow$.

4.1.5 Expansion Tensor:

$$\theta = U^i_j + U^i\Gamma^i_{ki}$$

$$\theta = -[\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}]$$

$\theta = -[n\frac{\dot{B}}{B} + \frac{2\dot{B}}{B}]$ put $n = -3/2$ then $\theta = -\frac{\dot{B}}{2B}$ (31)

$$\theta = -\frac{1}{2}\frac{\frac{D}{2}(t-)}{[\frac{D}{4}(t-)^2 - \frac{4}{5D}]}$$

Therefore, the Expansion Tensor tends to zero at $t \rightarrow$ and tends to zero at t is large.

4.1.6 Shear Scalar:

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\,\sigma^{ij}$$

Where $\sigma_{ij} = \frac{1}{2}\left[u_{i:k}u^k u_j - u_i u_j u^k\right] + \frac{1}{2}\left[u_{i:j} - u_{j:i}\right] - \frac{1}{3}\theta[g_{ij} - u_i u_j]$

$$\sigma^2 = \frac{1}{2}[\sigma_{11}\sigma^{11} + \sigma_{22}\sigma^{22} + \sigma_{33}\sigma^{33} + \sigma_{44}\sigma^{44}$$

$$\sigma^2 = \frac{7}{18}\theta^2 \quad (32)$$

$$\sigma^2 = -\frac{7}{18}\frac{D^2(t-)^2}{[\frac{D}{4}(t-)^2 - \frac{4}{5D}]^2}$$

Therefore, the Shear scalar tends to zero at $t \rightarrow$ and tends to zero at t is large.

4.1.7 Average Scale Factor:

$$V = R^3$$

$$R = V^{1/3} = (AB)^{1/3}$$

$$H_a = \frac{\dot{R}}{R}$$

$$H_a = \frac{1}{3}\left[\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right] = \frac{1}{3}\theta \quad (33)$$

$$H_a = -\frac{1}{12}\frac{D(t-)}{[\frac{D}{4}(t-)^2 - \frac{4}{5D}]}$$

Therefore, the Average Scale Factor tends to zero at $t \rightarrow$ and tends to zero at t is large.

4.1.8 The deceleration parameter q:

$$q = -\frac{\ddot{R}R}{\dot{R}^2}$$

Using $R = V^{1/3} = (AB)^{1/3}$ we get

$$q = -12\left[\frac{[\frac{D}{4}(t-)^2 - \frac{4}{5D}]}{D(t-)^2} + \frac{11}{2}\right] \quad (34)$$

Therefore, the deceleration parameter is negative at $t \rightarrow$ and t is large.

If the deceleration parameter shows a negative value then the model is inflated.

When

$$\lim_{t\rightarrow\infty}\frac{\sigma}{\theta} = \sqrt{\frac{7}{18}} \neq 0 \quad (35)$$

This means that the model is not isotropic.

4. 2 *Case II*

Solution of field equation:

In general relativity, Akarsu et al. (2012) and Berman (1983) proposed the FRW space-time field equation solution with linearly varying deceleration parameters. The Hubble parameter model of the universe uses the special law of variation as:

They have taken linearly varying deceleration parameters as:

$$q = -\frac{\ddot{R}R}{\dot{R}^2} = -\alpha t + m - 1 \ ,\alpha \geq 0 ,m \geq 0 \qquad (36)$$

The universe is decelerating when the deceleration parameter indicates a positive sign and the universe is accelerating when it indicates a negative sign. In the past the cosmological observations displayed the universe was decelerating but in the present time, the universe is accelerating. We consider the form of deceleration parameter as per the requirement from eq^n (47). We take $\alpha, m > 0$

$$B = c_2 \exp\{\frac{2}{\sqrt{m^2-2\alpha c_1}} \tanh^{-1}\frac{\alpha t-m}{\sqrt{m^2-2\alpha c_1}}\} \qquad (37)$$

Here c_1 & c_2 are arbitrary constants. Here the model is free from singularity because the scale factor B evolves exponentially. So, the pressure (P) and energy density (ρ) is found as:

Physical Properties:

4.2.1 Rest energy density:

From the eq^n (22)

$$\frac{2\dot{A}\dot{B}}{AB} - \frac{2\ddot{B}}{B} = \left(\rho - \frac{2J^2}{8\pi A^2B^2}\right)\frac{(8\pi+\lambda_2)}{\lambda_1}$$

$$2n\frac{\dot{B}^2}{B^2} - \frac{2\ddot{B}}{B} = \left(\rho - \frac{2J^2}{8\pi B^{2n+2}}\right)\frac{(8\pi+\lambda_2)}{\lambda_1}$$

Here $n \neq \pm 1$and putting B from the equation (25) we get

$$\rho = \frac{\lambda_1}{(8\pi+\lambda_2)}\left[\frac{8(n-1)\alpha^2-8\alpha^3t+8\alpha^2m}{(-2\alpha c_1-\alpha^2t^2+2\alpha tm)^2}\right] + \frac{H^2}{4\pi {c_2}^{2n+2}}\exp\{\frac{-4(n+1)}{\sqrt{m^2-2\alpha c_1}}\tanh^{-1}\frac{\alpha t-m}{\sqrt{m^2-2\alpha c_1}}\} \qquad (38)$$

If $m = \sqrt{2}, c_1 = -1, \alpha = -1, \lambda_1 = 1, \lambda_2 = \pi, n = -\frac{3}{2}, c_2 = 1, J = 1$ then the Rest Energy density of cloud string tends to constant at t is large.

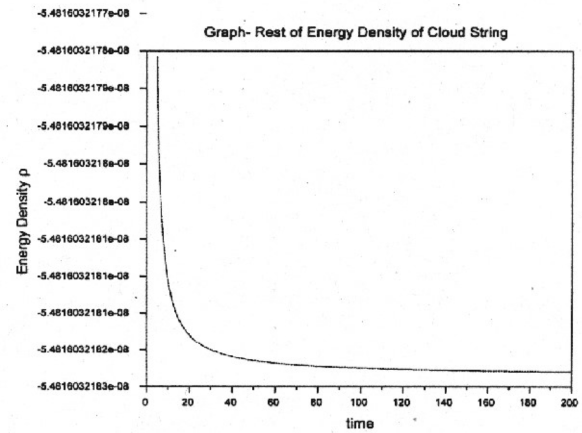


Fig. 3

4.2.2 Pressure P:

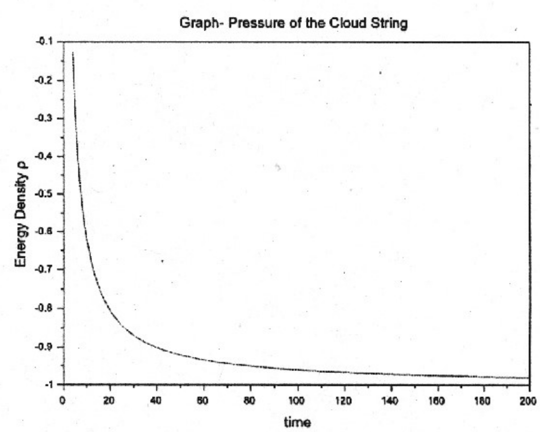
From the eq^n (17)

$$\frac{\dot{B}^2}{B^2} + \frac{2\ddot{B}}{B} + \frac{1}{B^2} = \lambda\frac{(8\pi+\lambda_2)}{\lambda_1} - \left(P - \frac{\lambda}{2} - \frac{\rho}{2}\right)\frac{\lambda_2}{\lambda_1}$$

Substitute the values of B from (25), λ from (27) and ρ from (28) we get

$$P = -\frac{(8\pi+3\lambda_2)}{\lambda_2}\frac{J^2}{4\pi {c_2}^{2n+2}}\exp\left\{\frac{-4(n+1)}{\sqrt{m^2-2\alpha c_1}}\tanh^{-1}\frac{\alpha t-m}{\sqrt{m^2-2\alpha c_1}}\right\} - \frac{\lambda_1}{\lambda_2}\left[\frac{6\alpha^2+8\alpha^3t-8\alpha^2m}{(-2\alpha c_1-\alpha^2t^2+2\alpha tm)^2}\right] + \frac{1}{c^2}\exp\{\frac{-4}{\sqrt{m^2-2\alpha c_1}}\tanh^{-1}\frac{\alpha t-m}{\sqrt{m^2-2\alpha c_1}}\} + \frac{\lambda_1}{(8\pi+\lambda_2)}\left[\frac{4(n-1)\alpha^2-4\alpha^3t+4\alpha^2m}{(-2\alpha c_1-\alpha^2t^2+2\alpha tm)^2}\right] \qquad (39)$$

If $m = \sqrt{2}, c_1 = -1, \alpha = -1, \lambda_1 = 1, \lambda_2 = \pi, n = -\frac{3}{2}, c_2 = 1, J = 1$then the Pressure of the cloud string tends to constant at t is large.



4.2.3 Spatial Volume:

From eq^n (30)

$$V = B^{1/2} \sin\theta$$

$$V = c^{1/2} \exp\{\frac{1}{\sqrt{m^2-2\alpha c_1}} \tanh^{-1} \frac{\alpha t-m}{\sqrt{m^2-2\alpha c_1}}\} \quad (40)$$

at $\theta = 90$

Therefore, the Spatial Volume is constant at t is large.

4.2.4 Expansion Tensor:

From eq^n (31)

$$\theta = -\frac{\dot{B}}{2B}$$

$$\theta = -\frac{\alpha}{(-2\alpha c_1-\alpha^2t^2+2\alpha tm)^2} \quad (41)$$

Therefore, the Expansion Tensor tends to be zero at t is large.

4.2.5 Average Scale Factor:

From eq^n (32)

$$H_a = \frac{1}{3}\left[\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right] = \frac{1}{3}\theta$$

$$H_a = -\frac{\alpha}{3(-2\alpha c_1-\alpha^2t^2+2\alpha tm)^2} \quad (42)$$

Therefore, the Average Scale factor tends to be zero at t is large.

4.2.6 Shear Scalar:

From eq^n (33)

$$\sigma^2 = \frac{4}{9}\theta^2$$

$$\sigma^2 = \frac{4}{9}\frac{\alpha^2}{(-2\alpha c_1-\alpha^2t^2+2\alpha tm)^4} \quad (43)$$

Therefore, the Shear Scalar tends to be zero at t is large.

5. Result & Discussion

It is observed that the following result of this model for two cases and plotted the graphs for a suitable choice of constants and discuss its physical properties.

Case I

- If $J \neq 0$ then the tensor density of the cloud of string tends to be constant at $t \rightarrow$ and if $J = 0$ then the tensor density is zero.
- The cloud string Rest energy density tends to be constant at $t \rightarrow$ and infinite at t is large.
- The Pressure of the cloud of string tends to be constant at $t \rightarrow$ and infinite at t is large.
- The Spatial Volume is constant at $t \rightarrow$ and infinite at t is large.
- The Expansion Tensor, Shear Scalar and the Average scalar factor tends to zero at $t \rightarrow$ and t is large.
- The deceleration parameter is negative at $t \rightarrow$ and t is large. Therefore, this model is inflated.

Case II

- The Rest of the energy density of the cloud string tends to constant at t is large.
- The Pressure of the cloud of string tends to constant at t is large.
- The Spatial Volume is constant at t is large.
- The Expansion Tensor, the Shear Scalar, and the Average scalar factor tend to zero at t is large.

Stability:

The model stability is dependent upon the function $c_s^2 = \frac{dP}{d\rho}$. If the function c_s^2 is greater than zero then the model is stable otherwise the model is unstable.

Case I

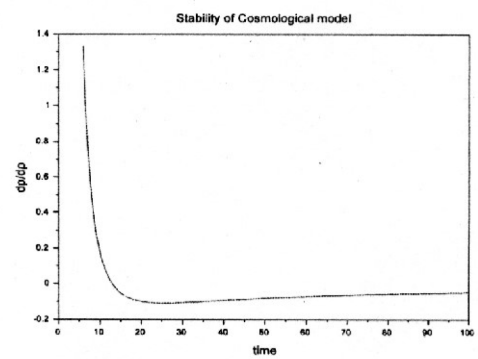


Fig. 5

Therefore, the value of the function c_s^2 is negative, so this model is unstable at t is large.

Case II

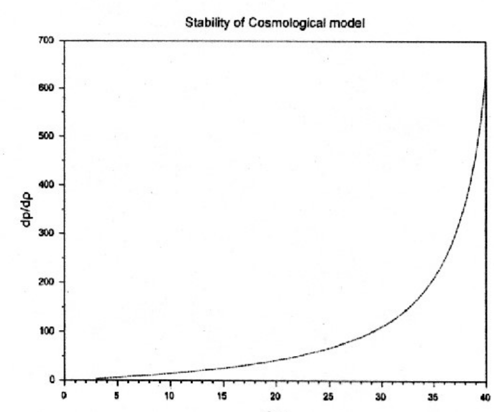


Fig. 6

Therefore, the value of the function c_s^2 is Positive, so this model is stable at t is large.

6. Conclusion

The fundamental of Kantowski-Sach Cosmological models are discussed earlier. In a nutshell, we have investigated a Kantowski-Sach string cosmological model in the electromagnetic field.

- Hubble parameter is positive from the beginning of the cosmic evolution and is a decreasing function at t is large. This shows the universe is only expanding.
- The Universe is not isotropic in the derived model at the present period because the ratio between the shear scalar and Expansion Tensor does not zero at t is large.
- The energy density of matter tends to zero at t is large in this model which is similar to the examination of Shaikh et al. (2021). The behaviour of the model is observed with the recent observational certainty of cosmology.
- At the initial epoch, The Universe starts with volume zero but in the Big-Bang scenario, expands exponentially approaching infinite volume.
- The deceleration parameter of the universe is negative at t is large; this sign shows the Universe is accelerating expansion.

The stability of the universe plays a major role in the string theory and the electromagnetic theory here we discuss. This is the novelty of its work. According to the cosmological explanations, In Case I, this cosmological model is unstable and In Case II, this model is stable at time is large according to the cosmological explanations. This is clear those cases are opposite to each other so this is very surprising result for the special case II this means Akarsu et al. (2012) and Berman (1983) condition is not correct for all types of cosmological modals. According to the above results case I solution is correct. This cosmological modal is unstable. The performance of the physical parameters is comparable to the obtained in the research paper Katore et al. (2016). Some other techniques are also check in future.

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